On the Survival of Short-Period Terrestrial Planets

Rosemary A. Mardling

Centre for Stellar and Planetary Astrophysics, School of Mathematical Sciences, Monash University, Victoria 3800, Australia

rosemary.mardling@sci.monash.edu.au

and

D. N. C. Lin

UCO/Lick Observatory, University of California, Santa Cruz, CA95064, USA

ABSTRACT

The currently feasible method of detection of Earth-mass planets is transit photometry, with detection probability decreasing with a planet's distance from the star. The existence or otherwise of short-period terrestrial planets will tell us much about the planet formation process, and such planets are likely to be detected first if they exist. Tidal forces are intense for short-period planets, and result in decay of the orbit on a timescale which depends on properties of the star as long as the orbit is circular. However, if an eccentric companion planet exists, orbital eccentricity (e_i, i for inner orbit) is induced and the decay timescale depends on properties of the short-period planet, reducing by a factor of order $10^5 e_i^2$ if it is terrestrial. Here we examine the influence companion planets have on the tidal and dynamical evolution of short-period planets with terrestrial structure, and show that the relativistic potential of the star is fundamental to their survival.

Subject headings: relativity — planetary systems — celestial mechanics — stars: late-type — stars: low-mass, brown dwarfs

1. Introduction

In the past few years, the radial velocity method has been used to detect more than 100 extrasolar planets (Kron et al. 2000) with minimum masses in the range 0.11 to 17 Jupiter masses. With the current lower detection limit of 3 m/s, discovering Earth-mass planets

with periods longer than a few days, this way is untenable. In principle, Earth-mass planets with periods less than 3 days may be marginally detectable with 1 m/s velocity precision (Narayan et al. 2004). But a large number of observations is needed to reduce the noise associated with the stellar radial velocity "jitter" (Saar et al. 1998).

Hopes for detecting Earth-mass planets in the near future lie with transit photometry and microlensing. So far, four gas giants have been observed crossing the faces of their stars (Brown et al. 2001; Konacki et al. 2003, 2004; Bouchy et al. 2004), three of which were discovered using microlensing techniques (Konacki et al. 2003, 2004; Bouchy et al. 2004). While the transit of a gas giant results in a maximum reduction of the star's brightness of around 1-2%, an Earth-mass planet would dim the star's light by only a few parts in 10,000. Nonetheless, such high-precision photometry is currently feasible in the MOST satellite and will be employed in the Kepler (Koch et al. 2001) mission. Simple geometrical arguments give the probability of seeing a planet cross the face of its parent star along our line of sight as $R_*/2a$, where R_* is the radius of the star and a is the distance of the planet from the star during the transit. Thus it is most likely that short-period Earth-like planets will be discovered first if they exist, especially since a weak signal can be strengthened by the integration of many orbits. This is not only true for planets discovered directly with transit photometry, but also those discovered using microlensing techniques since followup transit photometry is necessary for their confirmation and study.

In Section 2 we review the process of tidal damping, in Section 3 we discuss the dynamical effect of the presence of a companion, and in Section 4 we present a discussion.

2. Orbit Evolution due to Tidal Damping

During the formation epoch, tidal interactions between Jupiter-mass protoplanets and their nascent disks can cause them to spiral in toward their host stars (Goldreich and Tremaine 1980; Lin et al. 1986), a migration scenario (Lin et al. 1996) adopted to account for the origin of extrasolar planets with periods of a few days (Mayor and Queloz 1995). Other scenarios have been suggested (Rasio and Ford 1996; Nagasawa et al. 2003; Murray et al. 1998), all of which involve significant excitation of orbital eccentricity. Once a planet is in the vicinity of the star, however, tidal forces will tend to circularize the orbit as well as synchronize the planetary spin (Rasio et al. 1996). Earth-mass terrestrial planets may also be brought close to their host stars via one or more of these processes and their discovery (or otherwise) will shed much light on the planet formation process. For example, along their migration paths, gas giants induce residual terrestrial planets to undergo orbital evolution through their resonant and secular interaction. The detection of both short-period gas giants

and terrestrial planets around any host stars would provide overwhelming support for the core-accretion scenario.

Short-period planets are also capable of raising a substantial tide on their host stars. In order to avoid noise introduced by stellar jitters, the present planet-search campaigns focus on target stars with quiet chromosphere and slow spin rates. With one exception (Butler et al. 1997), all stars observed hosting short-period ("hot") Jupiters have spin periods considerably longer than the orbital period of the planet. Mature solar-type stars tend to be sub-synchronous because stellar winds carry away spin angular momentum through magnetic braking (Soderblom et al. 1993). Within these slowly spinning stars, the induced tidal oscillations act to dissipate their acquired tidal energy, the tidal response tends to lag the line of centers of the star and planet, and the resulting torque tends to spin up the star at the expense of orbital energy and angular momentum.

The simplest quantitative description for tides is the equilibrium tide model (Goldreich and Soter 1966; Hut 1981; Eggleton et al. 1998) which is adequate for analyzing the tidal response of terrestrial planets. In gaseous planets and stars, tidal perturbation can excite resonant responses which strongly enhance the dissipation rate (Zahn 1970). In radiative stars, the response is mostly in the form of g-mode oscillations whereas in fully convective envelopes of rapidly spinning gaseous planets and stars, it is in the form of inertial waves (Ogilvie and Lin 2004). Since we are primarily interested in the fate of the terrestrial planets in this paper, we adopt here the equilibrium tide model.

To leading order in the orbital eccentricity, the timescale, τ_a, for orbital decay is given by (Mardling and Lin 2002)

$$\frac{1}{\tau_a} = \left(\frac{\dot{a}_i}{a_i}\right)_{\rm star} + \left(\frac{\dot{a}_i}{a_i}\right)_{\rm planet}, \tag{1}$$

where

$$\left(\frac{\dot{a}_i}{a_i}\right)_{\rm star} = -\frac{9}{2}\left(\frac{n_i}{Q'_*}\right)\left(\frac{M_i}{M_*}\right)\left(\frac{R_*}{a_i}\right)^5\left[1-\left(\frac{P_{\rm orb}}{P_{\rm spin}}\right)\right] \tag{2}$$

and

$$\left(\frac{\dot{a}_i}{a_i}\right)_{\rm planet} = -\frac{171}{4}\left(\frac{n_i}{Q'_i}\right)\left(\frac{M_*}{M_i}\right)\left(\frac{R_i}{a_i}\right)^5 e_i^2 \tag{3}$$

are the contributions to orbital decay (or expansion) from tidal dissipation within the star and planet respectively, and the assumption has been made that the planet spins synchronously with the orbital motion. Here $P_{\rm orb}$ and $P_{\rm spin}$ are the orbital and stellar spin periods respectively, a_i is the semi-major axis of the orbit, n_i is the mean motion (orbital frequency), M_i, R_i, and Q'_i are respectively the mass, radius and *modified* Q-value of the planet, the latter

containing information about damping efficiency and rigidity (Goldreich and Soter 1966) (see Table 1), and M_*, R_*, and Q'_* are the corresponding values for the star. If the orbit is perfectly circular the planet makes no contribution to orbital shrinkage and the orbital decay timescale depends entirely on properties of the star. Otherwise the planet dominates the tidal decay process, and this is particularly true if it is Earth-like. In the case of slow stellar rotation and small eccentricity, the ratio of contributions to orbital decay from the planet and the star is $12.5e_i^2$ for a Sun-Jupiter pair, $1.3 \times 10^5 e_i^2$ for a Sun-Earth pair, and $2.2 \times 10^5 e_i^2$ for an M dwarf-Earth pair, where we have used Q'-values from Table 1. The discrepancy between the Jupiter system and the Earth systems is mostly due to the factor 10^4 difference in Q'-values of gas giants and terrestrials. Thus even a small orbital eccentricity allows a planet to dominate the tidal decay process if it has terrestrial structure, and can reduce the decay timescale considerably.

The timescale, τ_e, for eccentricity damping (or excitation) is given by an expression similar to Eqn. 1 (Goldreich and Soter 1966; Mardling and Lin 2002) except that it does not depend on e_i to leading order in e_i. Nonetheless, unless the star spins rapidly, the planet dominates the tidal circularization process (Dobbs-Dixon et al. 2004). Table 2 lists orbital decay and eccentricity damping timescales, together with other system parameters including transit probabilities for various real and hypothetical single planet systems. Included are the three of the shortest period planets discovered to date as well as HD209458. The values of a_i for the three hypothetical systems (Sun-Jupiter, Sun-Earth and M dwarf-Earth) were chosen to indicate how close to the host star we can ever expect to find planets in these types of systems. Note especially the steep dependence of τ_a on the eccentricity when the planet is Earth-like.

3. Eccentricity Excitation due to a Companion

HD209458 is included in Table 2 as one of only two transit systems discovered so far (Brown et al. 2001), the data for which allows an estimate of the planet radius of $1.35R_J$ which we use here to estimate timescales. The planet's relatively large size can be accounted for by inflation due to tidal dissipation of energy (Bodenheimer et al. 2001) if $e_i \sim 0.03$ and $Q'_i = 10^6$, with the corresponding orbital decay timescale being comparable to the age of the Universe. The observationally estimated eccentricity is small (< 0.05) but not zero (Fischer, private communication), even though τ_e is smaller than the estimated age of HD209458 (see Table 2). In order to resolve this paradox, the existence of a second planet has been conjectured (Bodenheimer et al. 2001) which secularly excites the eccentricity (Murray and Dermott 1999).

This eccentricity-excitation mechanism puts severe constraints on the existence of systems containing short-period terrestrial planets with companion planets. Figure 1a shows the evolution of the eccentricity of an Earth-mass planet in a 2.3 day orbit (a_i=0.02 AU) around an M dwarf of mass $0.2M_\odot$ which spins with a period of 40 days. It has a Jupiter-mass companion at a distance 0.7 AU which itself has an eccentricity of 0.2. The initial eccentricity of the inner planet is zero, however, it is excited by the presence of the companion to a maximum value which depends on the strength of additional perturbing accelerations which include the spin and tidal bulges of the star and planet, as well as the relativistic potential of the star (Mardling and Lin 2002).

In the absence of any of these accelerations, the inner eccentricity is excited to a value of 0.015 (the black curve). Taking half this value to represent the equilibrium inner eccentricity (see following paragraphs), the orbital decay timescale would be 7.1 Gyr. However, if all the perturbing accelerations listed above are included, the maximum inner eccentricity is only 0.0035, corresponding to an orbital decay timescale of 84.3 Gyr. This suggests that relativity plays a major role in the survival of short-period terrestrial planets with companions (given that Nature produces such systems in the first place). In fact it is the contribution it makes to the apsidal advance of the inner orbit which is responsible for the inhibition of the growth of the inner eccentricity (Holman et al. 1997). The fractional contribution the relativistic potential makes to apsidal advance compared to that made by a companion planet is given by $\gamma/(1+\gamma)$, where $\gamma = 4Z^2(M_*/M_o)(a_o/a_i)^3/(1-e_i^2)$, with M_o the mass of the companion planet and $Z = a_i n_i/c$ the ratio of the orbital speed of the inner orbit to the speed of light (Mardling and Lin 2002). For the case illustrated in Figure 1a, the relativistic potential is responsible for 95% of the apsidal advance of the inner orbit, while in the case of Mercury's orbit around the Sun, it contributes only 7%.

Even more extreme is the case in which a short-period terrestrial planet has an Earth-mass companion. For example, for a Sun-Earth-Earth system with semi-major axes 0.03 AU and 0.5 AU, the relativistic potential is responsible for 99.9% of the apsidal advance of the inner orbit. If the eccentricity of the outer planet is 0.3 the orbital decay timescale is 702 Gyr, while in the absence of relativity it would be only 0.47 Gyr.

Using equations which govern the secular evolution of the orbital elements for a dissipationless point-mass coplanar system (Mardling and Lin 2002; Murray and Dermott 1999; Wu and Goldreich 2002) and assuming small values for the eccentricities and a_i/a_o one can obtain an estimate for the variation of the inner eccentricity, δe_i, which also holds when the minimum eccentricity is zero at which point e_i is discontinuous as in Figure 1a. This estimate will vary slightly for moderately non-coplanar systems, and does not apply to resonant systems (Murray and Dermott 1999; Novak *et al.* 2003). If tidal dissipation is taken into

account, the system behaves as a damped autonomous system with the familiar circulatory and libratory behaviour (Fig. 1b). For a given set of initial conditions the system evolves to a fixed point corresponding to $\varpi_i - \varpi_o = 2n\pi$, where n is some integer and ϖ_i and ϖ_o are the longitudes of periastron of the inner and outer orbits respectively, and a finite equilibrium inner eccentricity, $e_i^{(\rm eq)}$, given approximately by

$$e_i^{(\rm eq)} = \frac{\frac{5}{8}(a_i/a_o)e_o}{|1-(M_i/M_o)\sqrt{a_i/a_o}+\gamma|} = \delta e_i/2. \tag{4}$$

Note that as well as the relativistic potential, γ may include contributions from other perturbing accelerations such as tidal and spin bulges. Figure 1c shows the dependence of $e_i^{(\rm eq)}$ on M_i/M_o in the absence of relativistic and other perturbing accelerations ($\gamma = 0$). The solid curves were obtained by integrating the Newtonian equations of motion for point masses (averaged over the inner orbit), while the dashed curves are given by Eqn. 4. Agreement is good except near the point corresponding to $M_i/M_o = \sqrt{a_o/a_i}$. The inclusion of relativistic effects introduces an additional scale which is proportional to m_i/a_i, and this is illustrated in Fig. 1d for the case $a_0/a_i = 10$. Only in one case is the discrepancy between the full solution and that given by Eqn. 4 evident.

Figure 2 consists of grey-scale plots of the orbital decay timescale of an Earth-mass planet orbiting a solar-mass star at 0.03 AU, as a function of the eccentricity, e_o, and semi major axis, a_o, of a Jupiter-mass perturber. The equilibrium eccentricity is calculated using the full governing equations (Mardling and Lin 2002), and this is then used in Eqn. 1 to obtain the decay timescale. The top panel is for the case where no perturbing accelerations are included, while the bottom panel includes the relativistic potential of the star. It is clear that relativity allows many systems to survive which would otherwise have suffered tidal destruction during the current lifetime of the star.

4. Discussion

Of particular interest amongst short-period terrestrial systems will be those with low-mass stars for which the habitable zone (planet surface temperature in the range 0-100^o) is at a distance where the transit probability is not negligible. Table 2 lists several hypothetical systems composed of a $0.2M_\odot$ M dwarf and an Earth-mass planet for which the habitable zone is around 0.04 AU. Planets in such close proximity to their host stars will be tidally locked so that one side of the planet is never directly heated. However, systems for which the habitable zone is further out may have planets locked in other spin-orbit resonances. Mercury is locked in a 3:2 spin-orbit resonance which relies on its permanent slight departure from sphericity as well as its substantially non-circular orbit (Goldreich and Peale 1966). Such

planets would be heated more evenly and hence be perhaps better candidates for detecting the signatures of life in their atmospheres.

While the Q-values of putative short-period terrestrial-type planets may turn out to be somewhat higher than the Earth's, either because of the absence of oceans or because of the different temperature profiles caused by the close proximity to the parent star, it is clear that a star's general relativistic potential must play a major role in the survival of such planets, and hence it is vital that it be included in any studies of this problem.

We thank A. Cumming, K. Freeman, G. Novak, and P. Sackett for useful discussions. This work was supported by the Victorian Partnership for Advanced computing Expertise Program Grant Scheme, and by NASA through NAGS5-11779 under its Origins program, JPL 1228184 under its SIM program, and NSF through AST-9987417.

REFERENCES

Bodenheimer, P. Laughlin, G. Lin, D.N.C. 2003, ApJ, submitted

Bodenheimer, P., Lin, D.N.C., & Mardling, R.A. 2001, ApJ, 548, 466

Bouchy, F., Pont, F., Santos, N. C., Melo, C., Mayor, M., Queloz, D. & Udry, S. 2004, A&A, 421, 13

Brown, T.M., Charbonneau, D., Gilliland, R.L., Noyes, R.W., & Burrows, A. 2001, ApJ, 552, 699

Butler, P.R., Marcy, G.W., Williams, E., Hauser, H. & Shirts, P. 1997, ApJ., 474, 115

Dickey, J.O. et al. 1994, Science, 265, 482

Dobbs-Dixon, I., Lin, D.N.C., & Mardling, R.A. 2004, ApJ, in press

Eggleton, P. E., Kiseleva, L. G. & Hut, P. 1998, ApJ, 499, 853

Goldreich, P. & Peale, S. 1966, AJ, 71, 425

Goldreich, P. & Soter, S. 1966, Icarus, 5, 375

Goldreich, P. & Tremaine, S. 1980, ApJ, 241, 425

Holman, M., Touma, J. & Tremaine, S. 1997, Nature, 386, 254

Hut, P. 1981, A&A, 99, 126

Koch, D. et al. & The Kepler Team 2001, Astronomische Gesellschaft Abstract Series, Vol. 18

Konacki, M., Torres, G., Sasselov, S.S., & Jha, S. 2003 Nature, 421, 507

Konacki, M, Torres, G, Sasselov, D. D., Pietrzynski, G., Udalski, A., Jha, S., Ruiz, M. T., Gieren, W. & Minniti, D. 2004, ApJ, 609, 37

Lin, D.N.C. & Papaloizou, J.C.B. 1986, ApJ, 309, 846

Lin, D.N.C., Bodenheimer, P., & Richardson, D.C. 1996, Nature, 380, 606

Marcy, G.W., Cochran, W.D., and Mayor, M. 2000, in Protostars and Planets IV, eds V. Mannings, A.P. Boss, & S.S. Russell, University of Arizona Press, Tucson, 1285

Mardling, R. & Lin, D.N.C. 2002, ApJ, 573, 829

Mayor, M. & Queloz, D. 1995, Nature, 378, 355

Murray, C.D. & Dermott, S.F. 1999, Solar System Dynamics, Cambridge University Press, Cambridge

Murray, N. Hansen, B., Holman, M., & Tremaine, S. 1998, Science, 279, 69

Nagasawa, M. Lin, D.N.C., & Ida, S. 2003, ApJ, 586, 1374

Narayan, R. Cumming, A. & Lin, D.N.C. 2004, ApJ, in preparation

Novak, G., Li, D., & Lin, D.N.C. 2003, ApJ, submitted

Ogilvie, G. & Lin, D.N.C. 2004, ApJ, in press

Rasio, F. A. & Ford, E. B. 1996, Science, 274, 954

Rasio, F.A., Tout, C.A., Lubow, S.H., & Livio, M. 1996, ApJ, 470, 1187

Saar, S.H., Butler, R.P., & Marcy, G.W. 1998, ApJ, 498, L153

Soderblom, D.R., Stauffer, J.P., Hudon, J.D., & Jones, B.F. 1993, Ap.J.Suppl., 85, 315

Wu, Y., & Goldreich, P. 2002, ApJ, 564, 1024

Yoder, C.F. 1995, Icarus., 117, 1

Yoder, C.F. 1995, in Global Earth Physics: A Handbook of Physical Constants, pp 1-31. Ed. T. Ahrens. American Geophysical Union, Washington DC

Yoder, C.F. & Peale, S.J. 1981, Icarus, 47, 1

Zahn, J.-P. 1966, Ann. Astrophys., 29, 313

This preprint was prepared with the AAS LaTeX macros v5.2.

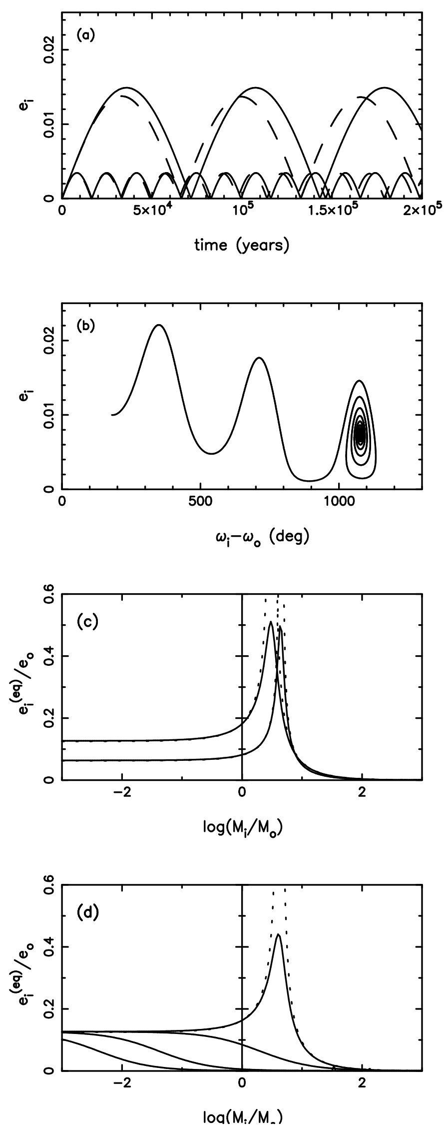

Fig. 1.— Eccentricity excitation by a companion planet. **a**, eccentricity evolution in the absence of tidal damping. $M_* = 0.2M_\odot$, $M_i = 1M_\oplus$, $M_o = 1M_J$, $a_i = 0.02$ AU, $a_o = 0.7$ AU, $e_o = 0.2$, and periastra initially aligned. Solid curves: top=no perturbing accelerations; bottom=relativistic potential only. Dashed curves: top=spin and tidal bulge only; bottom=both relativity and spin and tidal bulges. **b**, eccentricity evolution with tidal damping: circulatory and libratory behaviour in the $e_i - (\varpi_i - \varpi_o)$ plane. System parameters the same as in (a) for the case with no perturbing accelerations, but with initial $e_i = 0.01$, periastra antialigned, and an unrealistic $Q_i = 0.2$ used to illustrate the behaviour. In general, a two-planet system will evolve to a constant e_i with periastra aligned as long as τ_e is shorter than the lifetime of the system. **c**, dependence of $e_i^{(\mathrm{eq})}$ on M_i/M_o with $\gamma = 0$. Solid curves: Newtonian point mass equations; dashed curves: Eqn. (4). Top set: $a_o/a_i = 10$; bottom set: $a_o/a_i = 20$. **d**, dependence of $e_i^{(\mathrm{eq})}$ on M_i/M_o with relativistic effects included. $a_0/a_i = 10$. Bottom to top: (a_i/AU, m_i/M_J)=(0.05, 0.0033), (0.5, 0.0033), (0.05, 1), and (0.5,1).

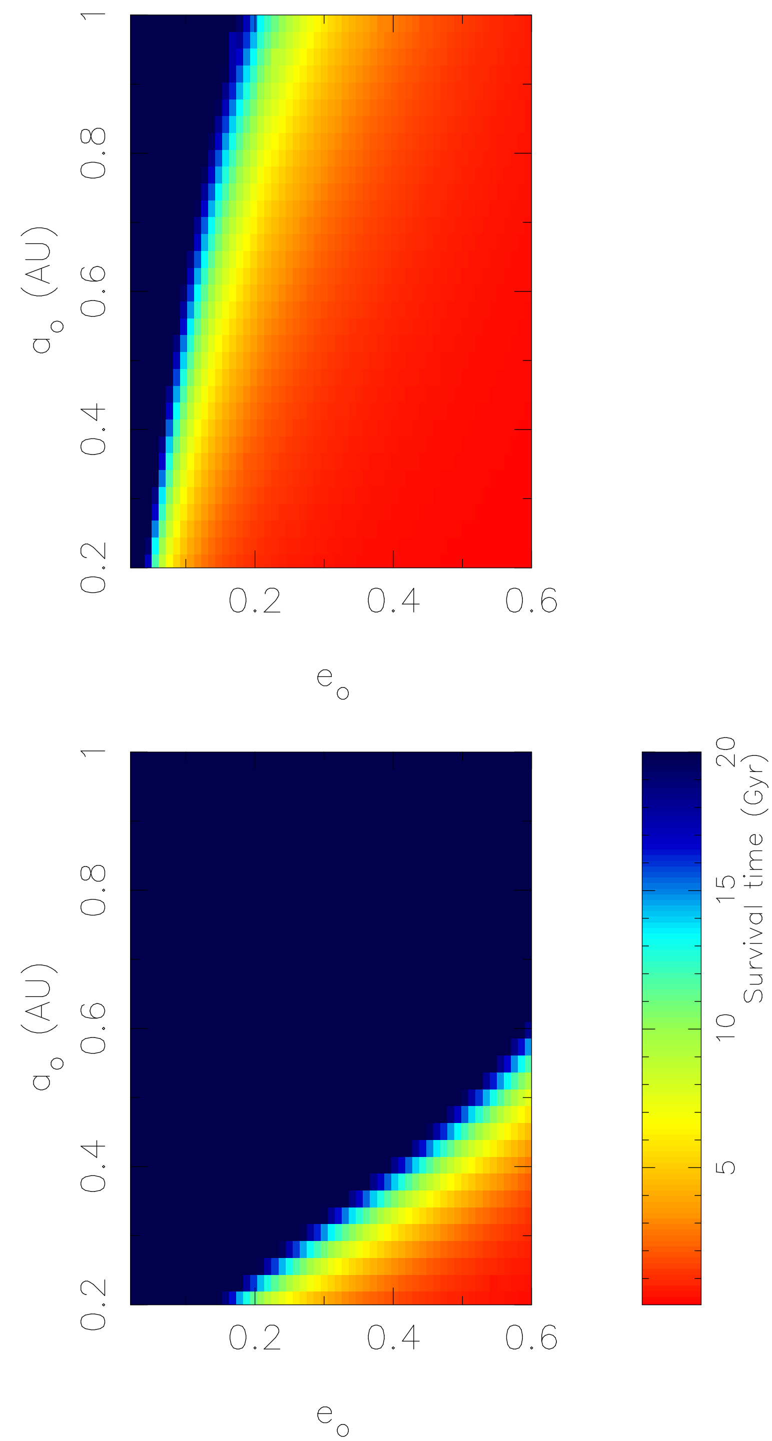

Fig. 2.— Orbital decay timescales with and without relativity as functions of the outer eccentricity, e_o, and semimajor axis, a_o. $M_* = 1M_\odot$, $M_i = 1M_\oplus$, $M_o = 1M_J$, $a_i = 0.03$ AU. Top panel: no relativity. Bottom panel: with relativity.

Table 1: **Structural parameters for various bodies**

Body	k_L	Q	Q'	reference
Venus	0.295	40	203.4	(1)
Earth	0.3	21.5	107.5	(2)
Mars	0.14	86	921	(3)
Jupiter	0.34	2.2×10^5	10^6	(4)
Sun	0.02	1.3×10^4	10^6	(5)
M dwarf	0.30	2.0×10^5	10^6	(5)

Love numbers (k_L), specific dissipation functions (Q), and modified Q-values (Q'), with $Q' = 3Q/2k_L$ (Goldreich & Soter, 1966, Murray & Dermott, 1999). Love numbers for gaseous bodies assume polytropic structure with polytropic indices 1, 3 and 1.5 for Jupiter, Sun and M dwarf respectively. References. (1) Yoder, 1995a, (2) Dickey et al., 1994, (3) Yoder, 1995b, (4) Yoder & Peale, 1981, (5) Mathieu, 1994.

Table 2: **Orbital decay timescales**

System	$M_*/M_\odot$	M_i/M_J	$P_{\rm spin}$	a_i (AU)	$P_{\rm orb}$	Pr	τ_e	$\tau_a(0)$	$\tau_a(0.01)$	$\tau_a(0.1)$
Sun/Jupiter	1.00	1.00	28	0.02	1.04 d	0.12	0.03	0.16	0.16	0.06
				0.03	1.91 d	0.08	0.45	2.34	2.30	0.88
				0.04	2.93 d	0.03	2.90	15.8	15.6	5.83
Sun/Earth	1.00	0.003	28	0.01	8.8 hr	0.23	2.2 − 5	0.53	0.009	8.1 − 5
				0.015	16.2 hr	0.15	3.0 − 4	7.41	0.12	0.001
				0.02	25.0 hr	0.12	0.002	48.7	0.78	0.007
				0.03	1.9 d	0.08	0.027	∞	10.9	0.10
				0.05	4.1 d	0.05	0.75	∞	∞	2.82
M dwarf/Earth	0.2	0.003	40	0.007	10.3 hr	0.10	2.4 − 5	0.97	0.01	8.9 − 5
				0.01	17.6 hr	0.07	2.4 − 4	9.88	0.097	9.0 − 4
				0.012	23.1 hr	0.06	7.9 − 4	34.4	0.32	0.003
				0.02	2.32 d	0.03	0.02	∞	8.82	0.082
				0.04	**6.56 d**	**0.01**	**1.97**	∞	∞	**7.39**
OGLE-TR-56	1.04	0.9	-	0.023	1.212 d	0.11	0.02	0.40	0.37	0.05
HD83443	0.79	0.41	36.5	0.038	2.986 d	0.05	2.1	73.2		
HD46375	1.00	0.25	-	0.041	3.024 d	0.06	5.1	26.7		
HD209458	**1.05**	**0.66**	**-**	**0.05**	**3.525 d**	**0.05**	**0.84**	**59.8**	**51.9**	**3.43**

Orbital decay and eccentricity damping timescales (Gyr), and transit probabilities (Pr), for various real and hypothetical single planet systems. M_i/M_J is the minimum mass of the planet in units of the mass of Jupiter, the actual mass being a factor $(\sin i)^{-1}$ higher when i is the inclination of the orbit to our line of sight, 'd' stands for days, and the argument of τ_a is the eccentricity. For all stars we took $Q'_* = 10^6$, for the known systems and the hypothetical Jupiter systems $Q'_i = 10^6$, while for the hypothetical Earth systems $Q'_i = 21.5$. The symbol ∞ corresponds to $\tau_a > 10^{11}$ yr. Entries in boldface for the last M dwarf-Earth system correspond to the approximate *habitable zone* for such a star.